THINKING STRATEGIES: DIVISION

BUILDING MASTERY OF DIVISION FACTS



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MathImagine



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INTRODUCTION

PROGRAM GOAL

Thinking Strategies: Division is a program designed to help students master the basic division facts. The program accomplishes this goal by (1) connecting the division facts to their related multiplication facts and (2) introducing the facts in logical rather than numerical order. The program uses the mathematical processes of communication, connections, reasoning, representation, and problem solving to encourage learning. Teachers can use *Thinking Strategies: Division* with an entire class, small groups, or individual students.

WHAT ARE THE BASIC DIVISION FACTS?

The basic division facts are combinations, like $40 \div 5$ or $27 \div 3$, in which the divisors (numbers doing the dividing) and the quotients (the answers) are both whole numbers less than 10. Because our number system is a base ten number system however, the program includes division facts in which the divisors and quotients are 10 as well as numbers less than 10.

WHAT IS MASTERY OF THE BASIC FACTS?

In his book, *Elementary and Middle School Mathematics*, John Van de Walle defines mastery of a basic fact as a quick response time of less than three seconds. This response should be accomplished without resorting to such inefficient means as counting. When a student is able to correctly respond to a fact automatically and without thinking, he/she has mastered that fact.

Building mastery of the basic facts involves the following four stages:

Stage 1: incorrect response/an inappropriate thinking strategy

Stage 2: correct response in more than three seconds using an appropriate thinking strategy

Stage 3: correct response within three seconds using an appropriate thinking strategy

Stage 4: correct response that is automatic and occurs without thinking

Mastery develops with practice (reviewing a variety of facts or procedures). The practice is provided in the program through student activity sheets, Power Facts, Partner Bingos, and Challenge Facts.

Mastering the Basic Facts: The Latest Research

Van de Walle states that all students are able to master the basic facts if they follow three steps:

- 1. develop a strong understanding of the operations and number relationships
- 2. develop efficient thinking strategies for fact retrieval
- 3. practice the use and selection of those strategies

WHAT IS A "THINKING STRATEGY"?

A thinking strategy is a way of thinking that helps complete a fact *quickly*. For a strategy to be a thinking strategy, it must be done *mentally*, and it must be efficient.

The more senses you can involve when introducing the facts, the greater the likelihood students will remember how to complete the facts. Different strategies work for different students. By providing a variety of strategies, students can choose what works best for them. Some strategies are visual – for example, the numbers of a clock, which can be used to complete facts with a divisor of 5. Some strategies are auditory and involve rhymes. Many of the strategies involve patterns and connecting facts that students have yet to learn with facts they already know how to complete. The primary thinking strategy for completing a division fact is to connect it to a related multiplication fact.

PROGRAM LEVELS

Thinking Strategies: Division is divided into seven levels. Because the primary thinking strategy for completing a division fact is to think of a related multiplication fact, each level begins by introducing the related multiplication facts. If students need more practice with these multiplication facts, teachers may want to refer to activities in the program *Thinking Strategies: Multiplication*. The order in which the facts are presented in the division program is the same as in the multiplication program.

In each level of *Thinking Strategies: Division*, students model the division facts introduced in that level. These models are visual representations of the facts and help students understand why a fact is completed the way it is. Next, thinking strategies for the division facts of that level of the program are introduced. Once the students have developed and understand these strategies, they practice them. These division facts are then incorporated with the division facts introduced in previous levels. At the end of each level, a Level Challenge activity helps students identify and apply thinking strategies to the division facts introduced to that point of the program.

LEVEL 1: In the first level, students examine the basic division facts with either a divisor or a quotient of 2. The concept of division and how the operation of division is related to the operation of multiplication are introduced.

LEVEL 2: In the second level, students examine the basic division facts with either a divisor or a quotient of 5. Students also examine division facts with either a divisor or a quotient of 10. Multiplication facts with factors of 5 and 10 and thinking strategies to complete these facts are introduced. Next, thinking strategies for division facts with either a divisor or a quotient of 5 or 10 are developed and practiced.

LEVEL 3: In the third level, students examine the basic division facts with either a divisor or a quotient of 9. These facts are introduced early in the program because of patterns that make them easier to complete.

LEVEL 4: In Level 4, students are introduced to division facts with either a divisor or a quotient of 1. As well, students are introduced to division facts with a quotient of 0. Students are also introduced to facts having the same divisors and quotients.

LEVEL 5: In the fifth level, students examine the basic division facts with either a divisor or a quotient of 4. There are only eight facts with either a divisor or a quotient of 4 that have not yet been introduced in the program.

LEVEL 6: In the sixth level, students examine the basic division facts with either a divisor or a quotient of 3. There are only six facts with either a divisor or a quotient of 3 that have not yet been introduced in the program.

LEVEL 7: In Level 7, students examine the basic division facts that have not yet been introduced in the program. There are only six facts that have not yet been introduced in the program. These facts have either a divisor or a quotient of 6, 7, or 8.

PROGRAM COMPONENTS

Teacher Lessons

The lessons give clear directions for working through the program and provide instructions that teachers can use to help students master the division facts. The lessons involve working with models, such as arrays and number lines. Students are encouraged to find as many strategies as possible for completing the facts. With many strategies to choose from, students soon find the strategies that work best for them.

Class Discussion

Class discussions are an integral part of the program. The following are some statements and questions that can be used to encourage student participation in class discussions:

Explain and justify your answer.

- Explain your answer in another way.
- Can someone else explain the answer in another way?

As a group, describe a different way to find the answer.

If you did not know how to complete this fact, what thinking strategy would you use?

What is another fact that you can complete this way?

How can you model this fact with a picture?

How can you model this fact with another picture?

How does this model explain the fact?

Make up a story problem for this fact.

How does your story problem illustrate the fact? Explain how these facts are related.

Practice for Students

The student activity sheets support understanding of number relationships and give students practice with the basic division facts. The activity sheets, which consist mainly of secret messages, line designs, and puzzles, are intended to be fun and engaging. They have the added advantage of being mainly self-correcting.

The program makes it easy to monitor the progress of students. A level challenge activity is at the end of every level. The Level Mystery Number Challenge checks the ability of each student to identify and apply the appropriate thinking strategies to the facts introduced to that point in the program. Teachers can keep track of each student's progress in other ways. As the students complete each level of the program, have them make a list of the facts they have yet to master from that level. Have the student practice these facts both at school with a classmate and at home with a parent. As the student is able to complete these facts, check them off on the student's progress report (page 163). While students are working on the activity sheet that accompanies each lesson, teachers can also check which facts students can complete.

At the completion of a level, students who have not yet mastered the facts introduced in that level can still continue to the next level of the program. They will have more opportunities to practice these facts in the levels that follow.

Power Facts

There are seven sets of Power Facts – one set for each level in the program. Each set has 20 facts. The facts consist of those that are introduced in that level plus some of the more difficult facts from previous levels. The Power Facts are asked only after the thinking strategies for those facts have been fully developed.

The Power Facts are intended to support the learning of the division facts. Ask these facts in class each day. Allow students no more than three seconds to complete each fact on their answer sheet, and then read out the division fact and its quotient (answer). If the students have been unable to complete the fact in three seconds, or if they have completed it incorrectly, have them write the completed fact on their answer sheet. Wait 5-7 seconds between facts to give the students time to process the fact. Students can measure their improvement each day as they are able to complete more facts correctly.

Each set of Power Facts is given in rows and columns. It is important to change the order in which the facts are asked. One day ask them vertically, the next day horizontally, then from top to bottom, and another day from bottom to top.

Multiplication/Division Grid

At the beginning of the program, hand out a copy of the multiplication/division grid to each student (page 248). In each level of the program, students are asked to fill in the grid for the facts they have been introduced to. Teachers may also want to keep a master grid.

Partner Bingo

Partner Bingo is a two-player game. Students can play Partner Bingo in class or at home with parents and siblings. Players have a bingo card and 16 facts that they take turns completing. As they complete each fact in order, they shade in one square on their bingo card. The same number might appear in more than one box on the bingo card, and the player must choose which to fill in. The first player to complete a row, column, or diagonal wins the game.

Partner Bingo provides practice with the basic division facts in a fun game setting. The games are a positive way for parents to determine which division facts their children are able to complete and which facts need more practice. There are Partner Bingo games in each level of the program.

Challenge Facts

The Challenge Facts (page 205) consist of division facts and some multiplication facts for each level of the program. Recent research does not support using timed tests to help students master the facts. Some students become very anxious when faced with timed tests, and this can affect their sense of their ability to do mathematics.

The Challenge Facts are included in the program as a diagnostic tool. Teachers can use these sheets to find out which facts the students have mastered and which facts they have not mastered. The students can circle the facts that require more practice on the Challenge Facts sheets. There is no time limit indicated on the Challenge Facts.

Students can use the Challenge Facts to practice identifying facts. To do this, students can use the same coloured pencil or draw the same shape around facts that have the same divisor or the same quotient.

Teacher Assessment

The intent of the program is not only to have the students master the basic facts, but also to have them do so in a positive manner. The evaluation and assessment should reflect this. Continuous assessment and evaluation allow recognition of student achievement. When students are successful in mastering the facts, they feel good about themselves and about their ability to learn. A powerful assessment tool is to have the students keep track of the facts that they have mastered. Doing so allows the students to see how much they are learning and to feel a sense of accomplishment.

The National Council of Teachers of Mathematics recommends the integration of assessment and instruction. The program *Thinking Strategies: Division* supports this integration. The student activity sheet that accompanies each lesson allows teachers to assess whether or not the students are following the program. As well, many of the activities are self-correcting and allow the students to know if they have understood the lessons.

Teachers can also use the Power Facts to assess the progress of students. Students can hand in their Power Facts answer sheets each day, or they can track their own progress. With either method, the teacher can use the results to complete the students' progress reports. Discourage students from guessing. Completing a fact incorrectly reinforces the incorrect answer.

Student Self-Assessment

In this program, students can track which facts they have mastered and which facts they have yet to master. As they complete each level of the program, have the students write about the facts introduced in that level in their journals. Ask them to indicate whether or not these facts are easy for them to complete. Have them explain why or why not. They may want to list both the facts they have mastered and the ones they have yet to master. Have them describe thinking strategies for the facts they have yet to master. If students need help in identifying thinking strategies, work with them to find the strategies. A student self-assessment progress report for each level is included in the program.

Using Individual Response Boards

Student response boards can be either a chalkboard or a dry-erase board made of laminated sheets of cardboard. In classrooms where each student has a response board, the students can write the answers to the facts on their boards. Teachers can check off the students' progress reports as the students show the answers on their boards.

Teachers in classes with no response boards can have students mouth the answer. This method of response provides students with the opportunity to answer a question, and it allows the teacher to monitor the progress of his/her students.

Parental Guidance

Teachers might want to involve parents in the learning process. Power Facts and thinking strategies can be sent home with students so that they can practice the facts with their parents as they are being taught the facts in class. Parents can also help their children by playing Partner Bingo with them.

MODELS USED IN THE PROGRAM

The following models are used in the program:

Groups of stars: The following groups of stars model represents the multiplication fact 3 x 2 and its related division facts 6 ÷ 2 and 6 ÷ 3. Unlike the operation of multiplication in which a groups of stars model represents one multiplication fact, groups of stars represent two division facts.



Number line: The following number line with 3 arrows or hops, each with a length of two, represents the multiplication fact 3 x 2 and its related division facts 6 ÷ 2 and 6 ÷ 3.



Arrays (boxes): A rectangular array is an arrangement of small boxes in rows and columns. Below, the array, consisting of boxes in 3 rows and 2 columns, represents the multiplication fact 3 x 2 and its related division fact 6 ÷ 3.

Ten-frame: A ten-frame is an array of 2 rows and 5 columns in which counters or dots are placed to illustrate numbers. The top row is filled in first, beginning on the left. Once the top row is full, the second row is filled in, again beginning on the left. For example, the multiplication fact 3 x 10 and its related division facts 30 ÷ 10 and 30 ÷ 3 can be represented by the following three ten-frames:

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۲	۲	۲	۲	۲
۲	۲	۲	۲	۲
۲	۲	۲	۲	۲
۲	۲	•	•	۲

Clock (for the division facts with a divisor of 5): The minute hand of a clock represents the number of minutes after the hour and can be used to represent division facts with a divisor of 5. The multiplication fact 3 x 5 and its related division fact 15 ÷ 5 can be represented by the following clock:



Buddies: One or two multiplication/division buddies appear on each student activity sheet. The buddies illustrate a fact introduced in that lesson of the program. The fact is usually the first fact of the activity sheet.



LANGUAGES IN DIVISION

Van de Walle explains that it is useful to think of number sentences, models, and word problems as three separate languages. For example, the fact 6 ÷ 2 can be expressed in the following ways:

- A number sentence is $6 \div 2 = 3$.
- A model could be an array of 2 rows and 3 columns.
- A word problem could be a story problem involving 6 pencils shared equally between 2 children.

When students complete a fact, having them translate from one language to another helps them develop operation meaning. Understanding the concept of division is the foundation for mastering and applying the division facts.

DIFFERENT CLASSES OF MULTIPLICATIVE STRUCTURES

Van de Walle explains that most researchers identify four categories of multiplication/division problems. These problems represent either a multiplication or a division problem, based on what is known and what is unknown. The four categories are: equal groups, multiplicative comparison, combinations, and product-of-measures. For each category, a multiplication problem and its two related division problems are given.

Equal-Groups Problem: Whole Unknown (Multiplication)

Jessica has 4 pencil cases. In each pencil case, she has 5 pencils. How many pencils does Jessica have in her pencil cases?

Equal-Groups Problem: Size of Groups Unknown (Division)

Jessica has 20 pencils. She wants to share the pencils equally among her 4 pencil cases. How many pencils should she place in each of her pencil cases?

Equal-Groups Problem: Number of Groups Unknown (Division)

Jessica has 20 pencils. She puts 4 pencils into each of her pencil cases. How many pencil cases does Jessica have?

Comparison Problem: Product Unknown (Multiplication) Danny lives 9 blocks from school. Ben lives 3 times as many blocks from school as Danny does. How many blocks from school does Ben live?

Comparison Problem: Set Size Unknown (Division)

Ben lives 27 blocks from school. He lives 3 times as many blocks from school as Danny does. How many blocks from school does Danny live?

Comparison Problem: Multiplier Unknown (Division)

Ben lives 27 blocks from school, and Danny lives 9 blocks from school. How many times as many blocks from school does Ben live compared to Danny?

 Combinations Problem: Product Unknown (Multiplication)

Sara has 7 T-shirts and 5 pairs of jeans. How many different outfits of T-shirts and jeans does Sara have?

Combinations Problem: Size of Set Unknown (Division)

Sara has a total of 35 different outfits consisting of T-shirts and jeans. If she has 7 T-shirts, how many pairs of jeans does she have?

Combinations Problem: Size of Set Unknown (Division)

Sara has a total of 35 different outfits consisting of T-shirts and jeans. If she has 5 pairs of jeans, how many T- shirts does she have?

 Product-of-Measures Problem: Product Unknown (Multiplication)

A room is 10 metres long and 9 metres wide. What is the area of the room?

Product-of-Measures Problem: Measure Unknown (Division)

The area of a room is 90 square metres. If the room is 10 metres long, what is the width of the room?

Product-of-Measures Problem: Measure Unknown (Division)

The area of a room is 90 square metres. If the room is 9 metres wide, what is the length of the room?

AND FINALLY...

Before you start the program, look through the seven levels. Explain to the students the order in which they will learn the division facts and why this order works (all facts are taught, and they are taught in logical order). Also discuss the goals of the program and the intended outcomes. Students can then understand where they are headed and what they can expect to achieve. You may also want to make a class poster of the thinking strategies.



LEVEL 1: FACTS WITH 2

LEVEL 1 OVERVIEW

In Level 1, students are introduced to the operation of division and its relationship to the operation of multiplication. The division facts in this level are facts with either a divisor or a quotient of 2. Division facts are closely tied to multiplication facts. Students who are able to complete their multiplication facts, are better able to complete the division facts. Level 1, therefore, begins by having students complete multiplication facts with a factor of 2.

Check to make sure your students are able to complete the multiplication facts with a factor of 2. If they need more practice with these facts, you may want to refer to the program *Thinking Strategies: Multiplication.* The first level of this program involves multiplication facts with a factor of 2. It introduces thinking strategies for completing these multiplication facts.

The program *Thinking Strategies: Division* reintroduces the thinking strategies for completing multiplication facts with a factor of 2. It then develops two thinking strategies for completing division facts with either a divisor or a quotient of 2: (1) the think-multiplication strategy and (2) the addition double strategy.

The think-multiplication strategy is the primary thinking strategy for completing all division facts. When applying the think-multiplication strategy, students are finding the missing factor of a related multiplication fact. For example, consider the fact $14 \div 2$.

14 ÷ 2

Think-Multiplication:

2 times what makes 14? 2 x 7 = 14 SO, 14 ÷ 2 = 7

OR

Think-Multiplication:

What times 2 makes 14? 7 x 2 = 14 SO, 14 ÷ 2 = 7

Either related multiplication fact can be used to complete the division fact. For reasons of efficiency, only one of the related multiplication facts is indicated in the program at a time.

The related division fact, $14 \div 7$, can be completed with the think-multiplication strategy as follows:

14 ÷ 7

Think-Multiplication:
 What times 7 makes 14?
 2 x 7 = 14
 SO, 14 ÷ 7 = 2

Addition Double Strategy

A division fact with a divisor of 2 can also be completed with the addition double strategy. For example, the fact $14 \div 2$ can be completed by finding the addition double that has a sum of 14:

14 ÷ 2

Addition Double:

What addition double has a sum of 14? 7 + 7 = 14 2 x 7 = 14 SO, 14 ÷ 2 = 7

The related division fact $14 \div 7$ can be completed by recognizing that the addition double 7 + 7 has a sum of 14 as follows:

14 ÷ 7

- Addition Double:
 7 + 7 = 14
 - 7 + 7 = 142 x 7 = 14 SO, 14 ÷ 7 = 2

Level 1 consists of the following lessons:

Lesson 1A: Zany Table with 2

In this lesson, the operations of multiplication and division are interpreted. The groups of stars model for both multiplication and division facts is introduced.

Lesson 1B: Zany Table with 2

This lesson discusses the commutative (turnaround) property of multiplication and introduces the rectangular array model. Fact families are also introduced.

Lesson 1C: Match-Ups with 2

This lesson checks whether or not the students have mastered the multiplication facts with a factor of 2. Thinking strategies for these multiplication facts are introduced.

Lesson 1D: Zany Table with 2

This lesson interprets the operation of division. It introduces the number line model for the operations of multiplication and division.

Lesson 1E: Zany Table with 2

This lesson introduces the think-multiplication strategy, which is the primary thinking strategy for completing division facts.

Lesson 1F: Wacky Webs with 2

In this lesson, students practice finding missing factors for multiplication facts with a factor of 2. The addition double strategy for facts with a divisor of 2 is introduced. This lesson also looks at even and odd numbers.

Lesson 1G: Cool Loops with 2

In this lesson, students are introduced to the addition double strategy for division facts with a quotient of 2.

Lesson 1H: Jellybean Jumble with 2

In this lesson, students practice the thinking strategies for division facts with either a divisor or quotient of 2.

Lesson 11: Secret Message with 2

In this lesson, students continue to practice the thinking strategies for facts introduced in this level. The ")—" symbol is also introduced.

Lesson 1J: The Level 1 Mystery Number Challenge

This challenge assesses the students' ability to identify and apply appropriate thinking strategies to the facts introduced in Level 1. Students also review models and vocabulary for the facts introduced in this level.

MULTIPLICATION/DIVISION GRID

The multiplication facts and their related division facts considered in this level are the following:

X	1	2	3	4	5	6	7	8	9	10
1		2								
2	2	4	6	8	10	12	14	16	18	20
3		6								
4		8								
5		10								
6		12								
7		14								
8		16								
9		18								
10		20								

Note: Since introducing division facts with a divisor of 0 is optional in the program, multiplication facts with a factor of 0 are not included in the multiplication/division grid.

LESSON 1A: ZANY TABLE WITH 2

TEACHER LESSON

Interpreting the Operation of Multiplication

Because the operation of division is related to the operation of multiplication, begin the program by presenting the students with a story problem that illustrates the multiplication fact 4 x 2. For example, consider the following story problem:

Leah has 4 scribblers. She puts 2 stars on each scribbler. How many stars has she put onto these scribblers altogether?

Have the students illustrate the story problem and solve it.

Note: This story problem illustrates the multiplicative structure of *equal groups*. Other multiplicative structures are possible and are discussed in the Introduction.



■ There are 8 stars altogether.

Ask the students to represent this story problem first as an addition sentence, and then as a multiplication sentence.

■ 2+2+2+2=8

Have the students note that each 2 represents 2 stars.

■ 4 x 2 = 8

Have the students note that the number 4 indicates the number of scribblers and the number 2 indicates the number of stars on each scribbler.

Explain that while the numbers in the addition sentence all represent stars, the numbers in the multiplication fact represent different things: the number 4 indicates the **number of groups** of stars; the number 2 indicates the number of stars in each group or the **size of each group**.

Have students describe other situations that model the fact 4×2 then draw pictures of these situations.

IMPORTANT! To make sure students understand multiplicative thinking, present a multiplication fact and have them create a story problem that involves the fact.

Modelling Facts

Groups of stars: Have students use groups of stars to model the multiplication fact 4×2 .

(**)(**)(**)(

Multiplication/division buddies: The multiplication/division buddy in this lesson shows 4 groups of 2.

Introducing the Operation of Division

Refer back to the story problem about Leah and her scribblers: Leah has 4 scribblers, and she puts 2 stars on each scribbler. Leah has 8 stars altogether. Now, explain to students that they are going to "turn the story problem around." There are two ways they can do this:

- Leah has 8 stars. She puts 2 stars on each scribbler. How many scribblers does Leah have? (Leah has 4 scribblers.)
- Leah has 8 stars. She wants to share the stars equally among her 4 scribblers. How many stars can she put on each scribbler? (Leah can put 2 stars onto each scribbler.)

Explain to students that both these story problems illustrate the operation of division. The first is represented by the division sentence $8 \div 2 = 4$. The second is represented by the division sentence $8 \div 4 = 2$. In both story problems, students are told the total number of stars and asked to separate or share these stars. Have the students compare these two story problems to the original, where they are asked to find the total number of stars.

The model for the multiplication sentence $4 \times 2 = 8$ illustrates both division sentences $8 \div 2 = 4$ and $8 \div 4 = 2$. Explain to students that these sentences are called *related number sentences*. The multiplication fact 4×2 has two related division sentences, $8 \div 2 = 4$ and $8 \div 4 = 2$.

The operation of division differs from the operation of multiplication in that most models only represent one multiplication fact but two division facts.

Note: The operation of division is considered in more detail in the following lessons. This lesson is only an introduction to the operation of division.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Zany Table, to each student. Read the instructions aloud as a class, and have the students complete the activity. The picture in the first row models the multiplication sentence $4 \times 2 = 8$. It also models two division sentences: $8 \div 4 = 2$ and $8 \div 2 = 4$. Below, complete the table row by row, then do what the sentence at the bottom of the page asks you to do.

Zany Table

*

(* (*

(* (*

THINKING STRATEGIES: DIVISION

*

Division Sentences	8 ÷ 4 = 2 8 ÷ 2 = 4				
Total Number of Stars	8				
Number of Stars in Each Group	2				
Number of Groups	4				
Multiplication Sentence	4 x 2 = 8		5 x 2 =		6 x 2 =
Picture	(* *) (* *) (* *)	$ \begin{array}{c} $		$\begin{pmatrix} \ast \\ \ast $	

Explain how the operations of multiplication and division are related.

LESSON 1B: ZANY TABLE WITH 2

TEACHER LESSON

Interpreting the Operation of Multiplication

Present students with the following story problem:

Three friends are at a birthday party. Each friend has 2 balloons. How many balloons do the friends have altogether?

Have the students illustrate the story problem and represent it as a multiplication sentence $(3 \times 2 = 6)$.



Now, present students with the following story problem:

Two friends are at a birthday party. Each friend has 3 balloons. How many balloons do the friends have altogether?

Have the students illustrate the story problem and represent it as a multiplication sentence $(2 \times 3 = 6)$.



Have the students note that the two multiplication sentences, $3 \times 2 = 6$ and $2 \times 3 = 6$, have the same product.

IMPORTANT! To help students with the concept of multiplication, say the fact 3 x 2 as "3 twos" rather than as "3 times 2." Say the fact 2 x 3 as "2 threes" rather than as "2 times 3."

Modelling Facts

Rectangular array model: The array that represents the multiplication fact 3×2 is an arrangement of 6 boxes with 3 rows and 2 columns. The array that represents the multiplication fact 2×3 is an arrangement of 6 boxes with 2 rows and 3 columns. The rectangular array is also a model for the operation of division. The first array represents the division fact $6 \div 3$, while the second array represents the division fact $6 \div 2$.



Note: The array model differs from most multiplication/division models as each array only represents one division fact rather than two.

Multiplication/Division buddies: The buddy in this lesson is holding an array that models the multiplication sentence $3 \times 2 = 6$ and the division sentence $6 \div 3 = 2$.

Introducing Terms for Multiplication

Tell students that in any multiplication fact, the numbers being multiplied are called the *factors* and the answer is called the *product*. In the multiplication fact 3×2 , the numbers 3 and 2 are called the *factors*, while the answer, 6, is called the *product*.

$$3 \times 2 = 6$$

factors product

The Commutative Property of Multiplication

Have the students consider the multiplication facts 3×2 and 2×3 . Although the first fact represents "3 twos" and the second fact represents "2 threes," each has the same product, 6.

Ask the students: Does the order of the factors in a multiplication fact affect its product?

 No. This is known as the turnaround or commutative property of multiplication.

To illustrate this property, have students consider the arrays modelling the multiplication facts 3×2 and 2×3 . Each array consists of 6 boxes. If the array modelling the multiplication fact 3×2 is turned on its side, it now represents the multiplication fact 2×3 .

IMPORTANT! The operation of division is not commutative. Consider the facts $6 \div 3$ and $3 \div 6$. The completed facts are $6 \div 3 = 2$ and $3 \div 6 = \frac{1}{2}$.

Introducing Fact Families

Write the multiplication sentences $3 \times 2 = 6$ and $2 \times 3 = 6$ on the chalkboard. Ask students to name the related division sentences for each ($6 \div 3 = 2$ and $6 \div 2 = 3$). Explain that these four facts are all related and form a *fact family*.

3 x 2 = 6 2 x 3 = 6 6 ÷ 3 = 2 6 ÷ 2 = 3

Have the students note that the product of the multiplication facts, 6, is the first number in each division fact.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Zany Table, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Zany Table



You can model a multiplication fact with an array. For example:

These arrays also model the related division sentences $6 \div 3 = 2$ and $6 \div 2 = 3$. The facts $3 \times 2 = 6$, $2 \times 3 = 6$, $6 \div 3 = 2$, and $6 \div 2 = 3$ form a fact family.

In the multiplication sentence $3 \times 2 = 6$, the numbers 3 and 2 are called the *factors* and the answer is called the *product*.

 $2 \times 3 = 6$

Complete the table below, then follow the instructions at the bottom of the page.

Multiplication Sentence	Picture	Related Multiplication Sentence	Picture	Related Division Sentences
3 x 2 = 6		2 x 3 = 6		
9 x 2 =		2 x 9 =		

Do the facts 3 x 2 and 2 x 3 have the same product? Explain.

Name the fact families:



LESSON 1C: MATCH-UPS WITH 2

TEACHER LESSON

Interpreting the Operation of Multiplication

Present students with the multiplication fact 2×9 . Have students explain what the fact 2×9 means to them. For example:

2 x 9 means 2 nines or 2 groups of 9 9 + 9 = 18 SO, 2 x 9 = 18

The two main thinking strategies for completing the fact 2×9 are the *addition double strategy* and the *helping fact strategy*.

Introducing the Addition Double Strategy for Multiplication

Remind students that an addition fact with the same addends, such as 9 + 9, is called a *double*. An effective thinking strategy for completing a multiplication fact with a factor of 2 is to connect it to its corresponding double fact.

Students can relate doubles to special pictures that provide visual cues. The following special pictures are used in the Thinking Strategies program:

- 3 + 3 = 6 a bug with 3 legs on each side
- 4 + 4 = 8 a spider with 4 legs on each side
- 5 + 5 = 10 a person with 5 fingers on each hand
- 6 + 6 = 12 an egg carton with 2 rows of 6 eggs
- 7 + 7 = 14 a calendar with 2 weeks of 7 days
- 8 + 8 = 16 a box of crayons with 2 rows of 8 crayons

9 + 9 = 18 - an 18-wheeler double with 9 wheels on each side

Introducing the Helping Fact Strategy

A helping fact has one factor in common with the fact being completed. Students can use any fact they have mastered to help them complete a fact that they have not yet mastered. Because our number system is a base ten number system, facts with a factor of 5 or 10 are often used as helping facts. A helping fact with a factor of 10 is particularly useful for completing facts with a factor of 9.

Applying Thinking Strategies for Multiplication

2 x 9

Addition Double:

The special picture for the addition fact 9 + 9is an 18-wheeler double. 9 + 9 = 18SO, $2 \times 9 = 18$ **Note:** Because of the turnaround or commutative property of multiplication, the fact 9 x 2 can also be completed by the addition double strategy. The addition double strategy can be applied to any multiplication fact with a factor of 2.

Helping Fact:
 2 x 10 = 20
 20 - 2 = 18
 SO, 2 x 9 = 18

Note: Although skip-counting is important when students are becoming familiar with the products of facts with a factor of 2, skip-counting has not been included as a thinking strategy. Encourage students to use other more efficient strategies to complete these facts.

Assessing Mastery of Multiplication Facts with a Factor of 2

When students are able to complete a multiplication fact with a factor of 2, they are better able to complete its related division facts. Use Challenge Facts 1 (page 206) to assess whether or not students have mastered the multiplication facts with a factor of 2.

IMPORTANT! Have students who have not yet mastered the multiplication facts with a factor of 2 complete the Student Activity sheets in Level 1 of the program *Thinking Strategies: Multiplication* (pages 7-23).

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Match-Ups, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Note: Students will find the connecting lines of each set of multiplication facts form a pattern. They can use the patterns to check their work.

COMPLETING THE GRID

Have the students fill in their multiplication grid for multiplication facts with a factor of 2. In the next lessons, they will develop thinking strategies for the related division facts.

Note: Students can fill in both a row and a column of the grid. Remind students that the grid illustrates the commutative property of multiplication.

Match-Ups

Complete each multiplication fact by connecting the dot beside the fact to the dot beside its product. Use a ruler to keep the lines straight.

When you have completed the "match-ups," explain your thinking strategy at the bottom of the page.

2 x 9	·CA	• 4	2 x 6	•	•	10
6 x 2	.OA	• 8	8 x 2	•	•	2
2 x 4	•	• 12	2 x 5	•	•	12
2 x 2	•	• 18	2 x 1	•	•	16
7 x 2	•	• 6	2 x 7	•	•	14
2 x 3	•	• 14	9 x 2	•	•	20
2 x 8	•	• 2	3 x 2	•	•	8
5 x 2	•	• 20	2 x 10	•	•	18
2 x 10	•	• 10	4 x 2	•	•	6
1 x 2	•	• 16	2 x 2	•	•	4

Explain what the fact 2 x 9 means.

Explain your thinking strategy for completing the fact 2×9 .

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LESSON 1D: ZANY TABLE WITH 2

TEACHER LESSON

Practicing Multiplication Facts

When students are able to complete a multiplication fact, the better able they are to complete its related division facts. Have the students complete multiplication facts with a factor of 2. Present six numbers to the students (or have them choose six numbers). Choose numbers less than or equal to 10. For example, consider the following numbers:

7	5	8	2	6	c
1	5	0	~	0	

Have students multiply each number by 2 as follows:

7	5	8	2	6	9
x 2	<u>x 2</u>				

Encourage the students to use the addition double or helping fact strategy to complete the facts.

Interpreting the Operation of Division

Present the division sentence $14 \div 2 = 7$ to the students. Have students create a story problem that illustrates the division sentence. You might suggest the story problem involves goldfish and goldfish bowls. Encourage the students to begin the story problem with 14 goldfish.

The division sentence $14 \div 2 = 7$ can be interpreted two ways. Some students will separate the 14 goldfish into 7 goldfish bowls, each containing 2 goldfish. These students are **separating** 14 into as many **groups of 2** as possible. In this interpretation, the size of each group is 2. The number of groups is 7. This interpretation is known as the **measuring** or **grouping** interpretation and can be modelled as follows:



Other students will share the 14 goldfish equally between 2 goldfish bowls. These students are sharing 14 equally between 2 groups. The size of each group is 7. This interpretation is known as the **partitioning** or **sharing** interpretation and can be modelled as follows:



Note: Although it is important that the students use and understand both interpretations of a division fact, it is not important they know the names of these interpretations.

Encourage students to use both interpretations when explaining why $14 \div 2 = 7$. Have them name the related multiplication facts for these interpretations. The related multiplication fact for separating 14 into as many groups of 2 as possible is $7 \times 2 = 14$. The related multiplication fact for partitioning 14 into 2 groups is $2 \times 7 = 14$.

The two interpretations of division can be challenging for students. Creating story problems that illustrate division facts can help students with these interpretations. The phrases "separate into groups of" and "share equally among" can also help. For example, consider the fact $14 \div 2$. The two interpretations of the fact $14 \div 2$ are "separate 14 into groups of 2" and "share 14 equally between 2 groups."

Introducing Terms for Division

Explain to the students that in any division fact, the number being separated is called the *dividend*. The number doing the separating is called the *divisor*, and the answer is called the *quotient*. For example, in the division sentence $6 \div 3 = 2$, the number 6 is the dividend, the number 3 is the divisor, and the answer 2 is the quotient.



Modelling Facts

Number Lines:

Students can model the two interpretations of the division sentence $14 \div 2 = 7$ with hops on number lines as follows:



The number of hops on the number line indicates the number of groups. The length of each hop indicates the size of each group.

Note: The numbers indicated on the number line are multiples of 5. The multiples of 5 serve as anchors in our base-ten number system.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Zany Table, to each student. Read the instructions aloud as a class, and have the students complete the activity.

		You can model the	division sentence $14 \div 2 = 7$ in two different way	s on a number line:
			$\begin{array}{c} 2 & 2 \\ 10 & 15 \\ 10 & 15 \\ 10 & 15 \\ 10 & 5 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ $	15 20
		For the division se and 7 is called the	ntence $14 \div 2 = 7$, 14 is called the <i>dividend</i> , 2 is <i>quotient</i> .	called the <i>divisor</i> ,
	10 15 20	Below, complete the page asks you to o	le table row by row, then do what the sentence alo.	at the bottom of the
Division Sentence	Pictur	ē	In Words	Multiplication Sentences
14 + 2 =		² 15 20	Separate 14 into groups of 2.	7 x 2 = 14
		15 20	Share 14 equally between 2 groups.	2 x 7 = 14
10 ÷ 2 =	$\overset{\bullet}{\underbrace{+}}_{0} \begin{array}{c} & & \\ & $	++++++++ 15 20		
	▲	++++++++ → 15 20		
		2 2 15 1 1 + + + + +		
	↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	15 15 20		
12 + 2 =	$\checkmark 10 \qquad 5 \qquad 10 \qquad 10$	++++++++ 15 20		
	$\overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}_{0} + \overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}}_{10} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}}_{10} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{10} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{10} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}{\overset{\circ}}}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}}}_{5} + \overset{\bullet}{\overset{\bullet}}_{5} + \overset{\bullet}{\overset{\bullet}{\overset{\circ}}}_{5} + \overset{\bullet}{\overset{\bullet}}_{5} + \overset{\bullet}{\overset{\bullet}}_{5}$	+++++++ >		
Explain what the divi	ision sentence 14 ÷ 2 mea	ns.		

Zany Table

LESSON 1E: ZANY TABLE WITH 2

TEACHER LESSON

Practicing Multiplication Facts

Continue to have the students complete multiplication facts with a factor of 2. One way to practice is to present students with nine of these facts and a tic-tac-toe grid (page 249). For example, consider the following facts:



Divide the class into two teams, or have the students play against the teacher. After each fact is completed, teams take turns placing an X or an 0 in one of the nine spaces. The first team to complete a row, column, or diagonal of Xs or 0s wins the game.

Interpreting the Operation of Division

Present the division fact $10 \div 2$ to the students. Have students explain what this fact means to them. Have the students model the fact with a picture then name its related multiplication sentence.

Remind students that there are two ways to represent the division fact $10 \div 2$ with groups of stars.



Separate 10 into groups of 2.

The related multiplication sentence is $5 \times 2 = 10$.



Share 10 equally between 2 groups.

The related multiplication sentence is $2 \times 5 = 10$.

Relating the Operations of Multiplication and Division

Have the students note that the quotient of the division fact, 5, is a factor of the multiplication facts. Therefore, students can find the quotient of the division fact $10 \div 2$ by finding a missing factor of a related multiplication sentence.

Introducing the Think-Multiplication Strategy

Because of the relationship between the operations of multiplication and division, students can complete a division fact by finding the missing factor of either of its related multiplication sentences. This strategy is called the *think-multiplication strategy* and is the primary thinking strategy for completing division facts. The think-multiplication strategy can be used to complete the division fact $10 \div 2$ as follows:

10 ÷ 2

Think-Multiplication:
 What times 2 makes 10?
 5 x 2 = 10
 SO, 10 ÷ 2 = 5

OR

Think-Multiplication:
 2 times what makes 10?
 2 x 5 = 10
 SO, 10 ÷ 2 = 5

Note: Either related multiplication sentence can be used. For reasons of efficiency, only one of the related multiplication sentences is used in the program at any one time. However, encourage the students to use either of the related multiplication sentences when they apply the think-multiplication strategy.

Creating Story Problems

Have students create a story problem that involves the fact $10 \div 2$ and model it with a picture. Suggest the story problem involves cars and passengers. Next, have students use the same picture, but change the story problem to one that involves the operation of multiplication. The relationship between the operations of multiplication and division is reinforced when students change a story problem illustrating a division fact to one illustrating a related multiplication fact.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Zany Table, to each student. Read the instructions aloud as a class, and have the students complete the activity.

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T

Explain how you can use a think-multiplication strategy to complete the division fact 12 ÷ 2.

LESSON 1F: WACKY WEBS WITH 2

TEACHER LESSON

Finding Missing Factors

Present the multiplication sentence $2 \times \square = 12$ to the students. Have the students note that the " \square " notation represents a missing factor. Have students find the missing factor of the multiplication sentence.

 $2 \times 6 = 12$ SO, the missing factor is 6.

Have the students note they can use the multiplication sentence $2 \times 5 = 10$ to help them find the missing factor (2 fives make 10, so 2 sixes make 12). Encourage students to use multiplication facts with factors of 5 and 10 to help them find the missing factors.

Skip-Counting by 2

Students can use skip-counting to find the missing factors of the above facts. Although skip-counting is not included in this program as a thinking strategy for completing division facts, it can be helpful when students are first learning division facts. Have students skip-count by 2 to 20 as they work through the first level of the program.

Introducing the Addition Double Strategy for a Divisor of 2

All division facts can be completed using the thinkmultiplication strategy. It is the primary thinking strategy for completing the division facts. However, there are other thinking strategies for completing division facts. Many of these strategies involve "reversing" the thinking strategies for related multiplication facts. For example, students can complete facts with a divisor of 2 by "reversing" the addition double strategy for multiplication.

Students can use the addition double strategy to find the missing factor 2×12 as follows:

2 x 🗌 = 12

Addition Double:

What addition double has a sum of 12? 6 + 6 = 12 $2 \times 6 = 12$ SO, the missing factor is 6.

Note: Because of the commutative or turnaround property of multiplication, students can use the addition double strategy whether or not the missing factor is the first or second factor of the fact.

Even Numbers

Have the students name even numbers that are less than or equal to 20:

• 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20

Ask the students: Why are these numbers even?

Consider a set of 12 cubes. Since 12 cubes can be evenly divided into 2 sets of 6 cubes, the number 12 is an even number. The products of multiplication facts with a factor of 2 are always even numbers.

Have the students note that the ending numerals of the even numbers are 0, 2, 4, 6, and 8.

Odd Numbers

Have the students name odd numbers that are less than or equal to 20:

■ 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19

Ask the students: Why are these numbers odd?

None of these numbers can be expressed as the product of a basic multiplication fact with a factor of 2. Therefore, none of these numbers can be evenly divided into 2 equal groups of whole numbers.

Have the students note that the ending numerals of the odd numbers are 1, 3, 5, 7, and 9.

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Wacky Webs, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Note: The coloured cells form a pattern that students can use to check their work. Students can colour the cells with crayons, coloured pencils, markers, or highlighters. Have students note that since each fact has a factor of 2, the products are all even and the outer cells are not shaded in.

Wacky Webs



The numbers ending in 0, 2, 4, 6, and 8 are **EVEN** numbers. The numbers ending in 1, 3, 5, 7, and 9 are **ODD** numbers.

To find each missing number, ask yourself: What times the number in the middle cell makes the number in the outer cell?

Colour all of the cells that have an **ODD** number.



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LESSON 1G: COOL LOOPS WITH 2

TEACHER LESSON

Practicing Multiplication Facts

Present multiplication facts with a factor of 2 to the students. Have students complete the facts and explain their thinking strategies.

Interpreting the Operation of Division

Present the division fact $16 \div 8$ to the students. Have them explain what the fact means to them. Have students model the fact and complete it.

Interpretation #1: Separate 16 into as many groups of 8 as possible. Since 8 + 8 = 16, 16 can be divided into 2 groups of 8. This interpretation can be modelled with the following number line:



Interpretation #2: Share 16 equally among 8 groups. In this interpretation, the size of each group is 2. This interpretation can be modelled with the following number line.



Practicing the Think-Multiplication Strategy

The primary strategy for completing a division fact is the think-multiplication strategy. Have the students complete the fact $16 \div 8$ using the think-multiplication strategy.

16 ÷ 8

Think-Multiplication:

What times 8 makes 16? 2 x 8 = 16

SO, 16 ÷ 8 = 2

Introducing the Addition Double Strategy for a Quotient of 2

Another thinking strategy for division facts with a quotient of 2 is the *addition double strategy*. Again, have the students consider the division fact $16 \div 8$.

Using interpretation #1, separate 16 into as many groups of 8 as possible. Since 8 + 8 = 16, 2 groups of 8 are possible. Remind students that the fact 8 + 8 is called a *double fact*.

16 ÷ 8

Addition Double:

8 + 8 = 16 2 x 8 = 16 SO, 16 ÷ 8 = 2

Note: When students skip-count by 8 (8, 16), they reach 16 on the second count.

Naming Division Facts with a Quotient of 2

Have students name division facts with a quotient of 2. They include the following facts:

2 ÷ 1 = 2		
4 ÷ 2 = 2		
6 ÷ 3 = 2		
8 ÷ 4 = 2		
10 ÷ 5 = 2		
12 ÷ 6 = 2		
14 ÷ 7 = 2		
16 ÷ 8 = 2		
18 ÷ 9 = 2		
$20 \div 10 = 2$		

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Cool Loops, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Note: The loops form a pattern that students can use to check their work.

POWER FACTS

Now that the students have developed the thinking strategies for division facts that have either a divisor or a quotient of 2, hand out the Power Facts for Level 1 (page 156) and the letter for parents/guardians (page 171) for students to take home. Have students practice the first set of Power Facts at least once a day. Students can practice in class or at home.

PARTNER BINGO

Students can practice the division facts with either a divisor or a quotient of 2 by playing Partner Bingo, Level 1 (pages 174-175). Have the students complete the facts in order and cross out only one square on their card for each fact. Partner Bingo can be played in class or at home.

Cool Loops

Below, circle each pair of numbers that has a quotient of 2. The numbers must be beside each other horizontally, vertically, or diagonally. When you are finished, solve the problems at the bottom of the page.

	16	10	5	14	7	6
	2	8	12	18	3	4
	1	6	14	2	9	2
	8	6	1	7	16	12
	4	14	3	8	10	6
	7	20	10	18	9	5
Name ten o	division fac	ts with a qu	uotient of 2			
Explain you	ur thinking s	strategy for	⁻ completing	g the fact 1	6 ÷ 8.	

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LESSON 1H: JELLYBEAN JUMBLE WITH 2

TEACHER LESSON

Practicing Multiplication Facts

Have students continue to practice completing the multiplication facts with a factor of 2.

Identifying and Applying Thinking Strategies

Present the following division facts to the students:

Have the students complete the facts. The students must be able to recognize and apply an appropriate thinking strategy for each fact then explain their thinking strategies.

(a) 4 ÷ 2

Think-Multiplication:

2 times what makes 4? 2 x 2 = 4 SO, 4 ÷ 2 = 2

Addition Double:

What addition double has a sum of 4? 2 + 2 = 4 $2 \times 2 = 4$ SO, $4 \div 2 = 2$

(b) 12 ÷ 6

- Think-Multiplication:
 What times 6 makes 12?
 2 x 6 = 12
 SO, 12 ÷ 6 = 2
- Addition Double:
 - 6 + 6 = 12 2 x 6 = 12 SO, 12 ÷ 6 = 2

(c) 20 ÷ 10

Think-Multiplication:
 What times 10 makes 20?
 2 x 10 = 20
 SO, 20 ÷ 10 = 2

Addition Double:
 10 + 10 = 20
 2 × 10 = 20

2 x 10 = 20 SO, 20 ÷ 10 = 2

(d) 16 ÷ 2

- Think-Multiplication:
 2 times what makes 16?
 2 x 8 = 16
 SO, 16 ÷ 2 = 8
- Addition Double:
 What addition double has a sum of 16?
 8 + 8 = 16
 2 x 8 = 16
 SO, 16 ÷ 2 = 8

(e) 10 ÷ 5

- Think-Multiplication:
 What times 5 makes 10?
 2 x 5 = 10
 SO, 10 ÷ 5 = 2
- Addition Double:

5 + 5 = 10 2 x 5 = 10 SO, 10 ÷ 5 = 2

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Jellybean Jumble, to each student. Read the instructions aloud as a class, and have the students complete the activity.

Note: The coloured jellybeans form a pattern that students can use to check their work.

STUDENT JOURNAL

Have students choose two facts, one with a divisor of 2 and one with a quotient of 2. In their journals, have them explain their thinking strategies for completing these facts.









LESSON 11: SECRET MESSAGE WITH 2

TEACHER LESSON

Preparing for the Level 1 Challenge

Encourage students to practice the first set of Power Facts (page 156). Students can also play Partner Bingo, Level 1 (pages 174-175), either in class or at home, to prepare for the Level 1 Mystery Number Challenge.

Practicing Multiplication Facts

Have the students continue to practice completing the multiplication facts with a factor of 2.

Introducing the ") " Symbol

Tell students that not only can division be indicated by the "÷" symbol, it can also be indicated by the ") " symbol.

For example, consider the division fact $6 \div 3$. It can also be indicated by $3\overline{)6}$. Explain to students that in the first form, $6 \div 3$, the dividend appears first and the divisor appears second. In the second form, $3\overline{)6}$, the divisor appears first and the dividend appears second.

Note: Both $6 \div 3$ and $3\overline{)6}$ are read as "6 divided by 3."

Practicing Thinking Strategies

Present the following division facts to the students:

3)	6
7)	14
20	0
Z)	2

Have students complete each fact and explain the thinking strategies they used for each.

(a) 3)6

Think-Multiplication:

What times 3 makes 6? 2 x 3 = 6 SO, 6 ÷ 3 = 2

Addition Double:

3 + 3 = 6 2 x 3 = 6 SO, 6 ÷ 3 = 2

(b) 7)14

Think-Multiplication:
 What times 7 makes 14?
 2 x 7 = 14
 SO, 14 ÷ 7 = 2

Addition Double:

7 + 7 = 14 2 x 7 = 14 SO, 14 ÷ 7 = 2

(c) 2)2

- Think-Multiplication:
 - 2 times what makes 2? 2 x 1 = 2 SO, 2 ÷ 2 = 1
- Addition Double:
 What addition double has a sum of 2?
 1 + 1 = 2
 2 x 1 = 2
 SO, 2 ÷ 2 = 1

Note: Have the students note that the dividend and divisor are equal. Thinking strategies for facts with equal dividends and divisors are considered in more detail in Level 4.

(d) 2)18

Think-Multiplication:

What times 2 makes 18? 9 x 2 = 18 SO, 18 ÷ 2 = 9

Have students note that they can use the fact 10×2 to help them find the missing factor (10 twos make 20, so 9 twos make 18).

Addition Double:

What addition double has a sum of 18? 9 + 9 = 18 2 x 9 = 18 SO, 18 ÷ 2 = 9

Encouraging Class Discussion

Engaging students in whole-class discussions is an integral part of the program. Prompts for encouraging class discussions can be found on page 2 of the Introduction.

INTRODUCING THE STUDENT ACTIVITY SHEET

Distribute a copy of the sheet, Secret Message, to each student. Read the instructions aloud as a class, and have the students complete the activity.

CHALLENGE FACTS

Students can use Challenge Facts 2–4 (pages 207-209) to practice the division facts in this level.

Complete the facts below. The first fact is completed: 3 x 2 = 6. Check the Code Key. The letter above 6 is I. The letter I is written in the first blank. The second fact is completed: $5 \times 2 = 10$.

Secret Message



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LESSON 1J: THE LEVEL 1 MYSTERY NUMBER CHALLENGE

The Level 1 Mystery Number Challenge involves the division facts introduced in Level 1.

Before students take the Level 1 Mystery Number Challenge, review the terms *dividend*, *divisor*, and *quotient*. Have the students explain the think-multiplication strategy and why it can be used to complete division facts. Have the students explain the addition double strategy for division facts with either a divisor or quotient of 2.

Next, have the students complete division facts with either a divisor or quotient of 2. Choose facts that the students find most challenging. The following are often difficult for students to complete:

Have the students discuss the thinking strategies they can use with these division facts.

INSTRUCTIONS FOR THE LEVEL 1 CHALLENGE

Distribute a copy of the challenge sheet to each student. Explain the challenge: There are nine facts at the top of the page. Below the facts, there are nine boxes with a clue in each box. Complete the facts, then match each completed fact with one of the clues, and place it in that box. Cross out each fact after it has been placed in its box. After each fact has been placed into the box with its matching clue, add the quotients of the three facts in each row and in each column to find the mystery number.

SUGGESTIONS FOR MORE PRACTICE

Students will have many more opportunities to practice the division facts of this level as they work through the program. However, if the students need more practice, consider the following suggestions:

- Check that students understand the concept of multiplication. If they need more practice with multiplicative thinking, provide them with everyday situations that involve multiplication.
- Present students with story problems involving the operations of both multiplication and division. Have them solve these problems and illustrate them.
- Have students model both multiplication and division facts with pictures.

- Have students skip-count by 2 from 2 to 20.
- Check that students have mastered the multiplication facts with a factor of 2. The primary thinking strategy for completing division facts is the think-multiplication strategy. When students are able to complete the multiplication facts, they are better able to complete the division facts. If they need more practice with multiplication facts with a factor of 2, refer to the activity sheets in Level 1 of the program *Thinking Strategies: Multiplication*.
- Identify the multiplication facts that students find difficult.
 Help them develop thinking strategies for these facts.
- Have students list the multiplication facts they find difficult, and encourage them to practice these facts both with a classmate at school and a parent at home.
- Check that students can apply the think-multiplication strategy.
- Check that students can apply the addition double strategy.
- Encourage students to practice the first set of Power Facts (page 156).
- Have students play Partner Bingo in class or at home. There are 2 Partner Bingo games for Level 1 (pages 174-175).

Level 1 Mystery Number Challenge

Complete the facts below. Then, read the clue in each box, and find the fact that matches it. Place each completed fact into the box with its matching clue. To find the MYSTERY number, add the quotients of the three facts in each row and the quotients of the three facts in each column.

2 =	16 ÷ 8 =	18 ÷ 2 =	$2 \div 2 = 12 \div 6$	=
	14 ÷ 2 =	10 ÷ 2 = 12 ÷ 2	2 = 6 ÷ 2 =	
This g	s fact can be modelled with roups of stars as follows:	This fact separates 8 into groups of 2.	This fact can be completed by thinking, "6 times what makes 12?"	=
	This fact has the least quotient.	This fact can be modelled with the following number line: 0 5 10	This fact has a dividend of 18.	=
	This fact shares 10 between 2 groups.	This fact has a quotient of 6.	This fact can be modelled with the following array:	=

= _____

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APPENDIX A

TEACHER RESOURCES



POWER FACTS: LEVEL 1

14 ÷ 2 = 7	10 ÷ 5 = 2	20 ÷ 10 = 2	4 ÷ 2 = 2
20 ÷ 2 = 10	6 ÷ 3 = 2	16 ÷ 8 = 2	12 ÷ 2 = 6
12 ÷ 6 = 2	18 ÷ 2 = 9	2 ÷ 2 = 1	8 ÷ 4 = 2
2 ÷ 1 = 2	14 ÷ 7 = 2	6 ÷ 2 = 3	16 ÷ 2 = 8
8 ÷ 2 = 4	16 ÷ 2 = 8	18 ÷ 9 = 2	10 ÷ 2 = 5

THINKING STRATEGIES

The primary thinking strategy for completing a basic division fact is think-multiplication. This strategy is listed as the first thinking strategy for completing all division facts. Level 1 introduces facts with either a divisor or a quotient of 2. These facts can be completed with either the think-multiplication strategy or the addition double strategy.

THINKING STRATEGIES: DIVISOR OF 2

Consider, for example, the fact 14 ÷ 2.

14 ÷ 2

Think-Multiplication:

2 times what makes 14? 2 x 7 = 14 SO, 14 ÷ 2 = 7

Note: The related multiplication fact 7 x 2 = 14 can also be used to complete the division fact $14 \div 2$. For efficiency, only one of the related multiplication facts is indicated in the program.

Addition Double:

What addition double has a sum of 14? 7 + 7 = 14 2 x 7 = 14 SO, 14 \div 2 = 7

THINKING STRATEGIES: QUOTIENT OF 2

Consider, for example, the fact 14 ÷ 7.

14 ÷ 7

Think-Multiplication:

What times 7 makes 14? 2 x 7 = 14 SO, 14 ÷ 7 = 2

Addition Double:

7 + 7 = 14 2 x 7 = 14 SO, 14 ÷ 7 = 2



SELF-ASSESSMENT PROGRESS REPORT FOR STUDENTS • FACTS WITH 2

APPENDIX A

APPENDIX B

PARTNER BINGO





PARTNER BINGO 1

Find a partner.

Partner #1: Complete the first fact beside your card. On your card, find a square that matches the quotient of the first fact, and shade it in.

Partner #2: Complete the first fact beside your card. On your card, find a square that matches the quotient of the first fact, and shade it in.

Take turns completing the facts in order and shading in a square on your card. The first one to fill in a row, column, or diagonal wins the game.

Partner #1		Partne	er #1			
1) 16 ÷ 2 =	(9) 14 ÷ 2 =		1			
② 10 ÷ 2 =	10 16 ÷ 8 =	1	3	7	2	3
③ 18 ÷ 2 =	(11) 2 ÷ 2 =	5	2	9	6	5
④ 14 ÷ 2 =	(12) 8 ÷ 4 =		10		7	
(5) 2 ÷ 2 =	(13) 10 ÷ 2 =	1	10	÷	/	
⑥ 12 ÷ 6 =	(14) 18 ÷ 2 =	9	8	4	3	4
⑦ 16 ÷ 2 =	(15) 4 ÷ 2 =	2	2	10	Q	2
(8) 8 ÷ 2 =	(16) 12 ÷ 2 =	5	2	10	0	2
© 0 · 2	···		•			
Partner #2	U	Partne	er #2	-		
 B • • 2 Partner #2 1 12 ÷ 2 = 	 9 16 ÷ 2 = 	Partne	er #2			 1
 Partner #2 1 12 ÷ 2 = 6 ÷ 2 = 	 9 16 ÷ 2 = 10 6 ÷ 3 = 	Partne	er #2 2	1	2	10
 Partner #2 1 12 ÷ 2 = 6 ÷ 2 = 20 ÷ 2 = 	 9 16 ÷ 2 = 10 6 ÷ 3 = 11 8 ÷ 2 = 	Partne	er #2 2 5	1 7	2	10 9
Partner #2 1 $12 \div 2 = $ 2 $6 \div 2 =$ 3 $20 \div 2 =$ 4 $10 \div 5 =$	(9) $16 \div 2 = $ (10) $6 \div 3 = $ (11) $8 \div 2 = $ (12) $12 \div 2 = $	Partne	er #2 2 5	1 7	2	10 9
Partner #2 (1) $12 \div 2 = $ (2) $6 \div 2 = $ (3) $20 \div 2 = $ (4) $10 \div 5 = $ (5) $16 \div 2 = $	(9) $16 \div 2 = $ (10) $6 \div 3 = $ (11) $8 \div 2 = $ (12) $12 \div 2 = $ (13) $14 \div 7 = $	Partne 10 3 5	er #2 2 5 10	1 7 ÷	2 1 4	10 9 3
\bigcirc 0 · 2 - Partner #2 1 12 ÷ 2 = 2 6 ÷ 2 = 3 20 ÷ 2 = 4 10 ÷ 5 = 5 16 ÷ 2 = 6 8 ÷ 2 =	(9) $16 \div 2 =$ (10) $6 \div 3 =$ (11) $8 \div 2 =$ (12) $12 \div 2 =$ (13) $14 \div 7 =$ (14) $6 \div 2 =$	Partne 10 3 5 6	er #2 2 5 10 1	1 7 ÷ 2	2 1 4 6	10 9 3 8
Partner #2 (1) $12 \div 2 = $ (2) $6 \div 2 = $ (3) $20 \div 2 = $ (4) $10 \div 5 = $ (5) $16 \div 2 = $ (6) $8 \div 2 = $ (7) $18 \div 9 = $	(9) $16 \div 2 = $ (10) $6 \div 3 = $ (11) $8 \div 2 = $ (12) $12 \div 2 = $ (13) $14 \div 7 = $ (14) $6 \div 2 = $ (15) $20 \div 2 = $	Partne 10 3 5 6	er #2 2 5 10 1	1 7 ÷ 2	2 1 4 6	10 9 3 8

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PARTNER BINGO 2

Find a partner.

Partner #1: Complete the first fact beside your card. On your card, find a square that matches the quotient of the first fact, and shade it in.

Partner #2: Complete the first fact beside your card. On your card, find a square that matches the quotient of the first fact, and shade it in.

Take turns completing the facts in order and shading in a square on your card. The first one to fill in a row, column, or diagonal wins the game.



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APPENDIX C

CHALLENGE FACTS



LEVEL 1: FACTS WITH 2

Challenge Facts 1

Complete these facts as fast as you can.



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Complete these facts as fast as you can. = 16 = 14 = 20 x 2 = 12 x 2 = 6 x 2 = 4 7× 2 × × 8 = 10 = 16 = 10 20 x 2 = 18 x 2 = 2 П 2 X 5 × 2 × 8 8 18 x 2 = 12 2 = 8 2 ∥ x 2 = 6 4 П Ш ~ × × 9 x 2 X = 20 = 18 = 14 x 2 = 16 x 2 = 10 ∞ П 10 X 2 x 4 × 2 x = 14 12 x 2 = 16 ∞ ဖ 2 П П П Ш х Х 2 X 2 × 6 × 2 ×

LEVEL 1: FACTS WITH 2

LEVEL 1: FACTS WITH 2

Challenge Facts 3

Complete these facts as fast as you can.



WITH	
FACTS	
EVEL	

Challenge Facts 4 Complete these facts as fast as you can.

					S		
2)10	2)18	2)12	7)14	2)4	2)16	9)18	2)8
8)16	2)14	2)20	2)18	2)7	6)12	2)6	2)16
2)12	2)10	2)14	4)8	8)16	2)6	2)18	5)10
2)8	2)2	9)18	2)16	2)14	2)4	5)10	2)12
7)14	2)6	2)2	2)12	2)18	3)6	8)16	2)14

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APPENDIX E

ANSWER KEYS



Picture	Multiplication Sentence	Number of Groups	Number of Stars in Each Group	Total Number of Stars	Division Sentences
**	4 x 2 = 8	4	2	8	8 + 4 = 2 8 + 2 = 4
** ** ** ** ** **	7 x 2 = 14	7	N	14	14 + 7 = 2 14 + 2 = 7
** ** ** **	5 x 2 = 10	2	2	10	10 + 5 = 2 10 + 2 = 5
**************************************	8 x 2 = 16	8	2	16	16 + 8 = 2 16 + 2 = 8
(*) (*) (*) (*) (*) (*) (*) (*) (*) (*)	6 x 2 = 12	9	2	12	12 + 6 = 2 12 + 2 = 6
xplain how the operations of multiplicatio	on and division are	related.			
xplanations will vary.					

Lesson 1A, page 11



Lesson 1C, page 15

Multiplication Sentence	Picture	Related Multiplication Sentence	Picture	Related Division Sentences
3 x 2 = 6		2 x 3 = 6		6 ÷ 3 = 2 6 ÷ 2 = 3
7 x 2 = 14		2 x 7 = 14		14 ÷ 7 = 2 14 ÷ 2 = 7
9 x 2 = 18		2 x 9 = 18		16 ÷ 8 = 2 16 ÷ 2 = 8

Do the facts 3 x 2 and 2 x 3 have the same product? Explain.

Yes. The arrays representing the two facts have the same number of boxes.









Division Fact	Think- Muthinication	Picture	Multiplication	Division
10 + 2	2 times what makes 10?	Pictures will vary.	2 x 5 = 10	10 + 2 = 5
14 + 2	2 times what makes 14?	Pictures will vary.	2 x 7 = 14	
20 + 2	2 times what makes 20?	Pictures will vary.	2 x 10 = 20	
18 + 2	2 times what makes 18?	Pictures will vary.	9 x 2 = 18	
Explain how you can You can think 2 time	use a think-multiplic: what makes 12. Sir.	ation strategy to complete the division fact $12 + 2$.		

Lesson 1E, page 19



Lesson 1G, page 23









STUDENT ACTIVITY SHEETS

2	Vhy is an extra ", □	4" nice to have?	₫	L A L	
	5		. (6)		
	- 19 -	10 10 10 10 10 10	P	A C E 28 28 24	
	Code O A Key 1 2	3 S C L L	I E P 6 7 8	C T R U 9 10 14 16	- 0
0 3 x 2 = 6	2 5 x 2 = 10	8 2 x 5 = 10	● 8 x 2 = 16	B) 2 x 7 = 14	6 2 x 2 = 4
7) 2) <u>6</u>	 9)18 	(g) 2)16	10 2)10	$\underline{(1)} \ 6 \overline{)12}$	(12) 2)18
s 2)14	(a) 2)12	(1 €) 2)8	16 2)20	$\widehat{\mathbf{m}} \ 2\widehat{\mathbf{l}2}$	<u>18</u> 5)10
9 2) <u>16</u>	20 7 <u>114</u>	21 2)10	22 8) <u>16</u>	23) 2)18	2 € 2)14

Lesson 11, page 27



Lesson 2A, page 35

8 ÷ 2 = 4	16 ÷ 8 = 2	18 ÷ 2 = 9	2 ÷ 2 = 1 12 ÷ 6	6 = 2
1	4 ÷ 2 = 7	10 ÷ 2 = 5 12 ÷ 2	2 = 6 6 ÷ 2 = 3	
This fa grou	ct can be modelled with ps of stars as follows:	This fact separates 8 into groups of 2.	This fact can be completed by thinking, "6 times what makes 12?"	
	******* 14 ÷ 2 = 7	8 ÷ 2 = 4	12 ÷ 6 = 2	= <u>13</u>
t	This fact has he least quotient.	This fact can be modelled with the following number line: ++++++++++++++++++++++++++++++++++++	This fact has a dividend of 18.	= 13
	2 ÷ 2 = 7	6 ÷ 2 = 3	18 ÷ 2 = 9	
10	This fact shares between 2 groups.	This fact has a quotient of 6.	This fact can be modelled with the following array:	
	10 ÷ 2 = 5	12 ÷ 2 = 6	16 ÷ 8 = 2	= <u>13</u>
	= <u>13</u>	= <u>13</u>	= <u>13</u>	

Lesson 1J, page 29

Multiplication Sentence	In Words	Related Division Sentence	Related Division Sentence
3 x 10 = 30	3 groups of 10 make 30	30 + 10 = 3	30 ÷ 3 = 10
7 x 10 = 70	7 groups of 10 make 70	70 ÷ 10 = 7	70 ÷ 7 = 10
5 x 10 = 50	5 groups of 10 make 50	50 ÷ 10 = 5	50 ÷ 5 = 10
8 x 10 = 80	8 groups of 10 make 80	80 ÷ 10 = 8	80 ÷ 8 = 10
1 x 10 = 10	1 group of 10 makes 10	10 ÷ 10 = 1	10 ÷ 1 = 10
plain your thinking s	strategy for completing the div	vision fact 30 ÷ 10.	

Lesson 2B, page 37